CHAPTER II

FUNDAMENTALS OF HEAT TRANSMISSION

1. Transmission of heat from a solid body to air

If an air stream flows along the surface of a solid body, individual molecules of air collide with the solid surface and thus heat is conveyed from the surface to the air stream. In order to explain this phenomenon, let us examine the character of an air stream along the solid surface. [20]

Heat transfer is a very complex process and its seemingly simple expression by the Newtonian equation does not give results corresponding to those obtained by practical measurements.

The coefficient of heat transfer from a wall surface to a gaseous medium is a function of many variables, such as the shape and dimensions of the surface, direction and velocity of gas flow, temperature, density, viscosity, specific heat and thermal conductivity of the gas.

Differential equations expressing heat transfer have been successfully solved for extremely simplified assumptions only. Empirical methods have supplied no satisfactory results either, because of the existence of many variables, and generally valid relationships have been almost impossible to establish. Today the principle of similitude is the most reliable method by which to treat and generally apply experimental data on heat transfer by convection.

In theoretical studies viscosity of the medium (air) is not considered and velocity of the air stream is assumed to be equal throughout the whole cross section of the stream. Due to the viscosity of air, however, velocity decreases in the direction towards the solid surface till it becomes zero at the interface. (Fig. 28).

Thus a thin boundary layer is formed on the solid surface in which velocity is rising from zero up to the value of v i.e. the mean velocity of the surrounding medium. Fig. 29 shows the distribution of velocities at various distances x from the solid surface. The velocity distribution curve (velocity profile) is obtained by plotting actual velocities v in the boundary layer against distance x from the wall surface.

The shear stress between two adjacent layers can be expressed by an equation based upon Newton's work on internal friction. Thus:

$$\tau = \eta \frac{dv}{dx}$$

where τ denotes the tangential force (viscous shear)

η „ „ coefficient of viscosity

$\frac{dv}{dx}$ „ „ velocity gradient

Viscosity, as a property representing the reluctance of the medium to yield to tangential forces acting between adjacent layers, has been explained theore-

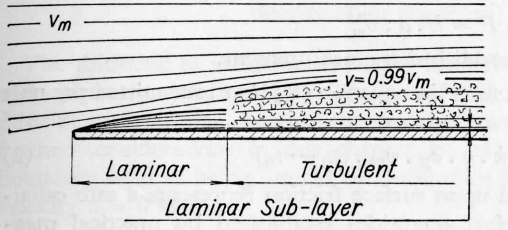

FIG. 28. Representation of the boundary layer on the surface of a solid body.

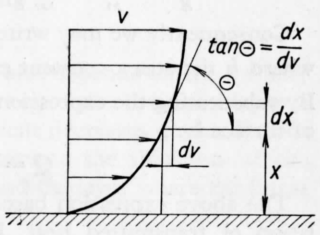

FIG. 29. Velocity profile of the boundary layer.

tically by the momentum transfer of molecules passing from one layer to another. Viscosity is, thus, physically a dissipation process of the same nature as heat convection; both are based on movements of molecules between two adjacent layers.

If the flow in the boundary layer is characterized by individual layers of the medium sliding one over the other at different velocities, in planes almost parallel to the solid surface, and with negligible transfer of molecules between the layers, we have the case of a laminar flow. In this case, heat transfer from one layer into the adjacent one, in a direction normal to the flow, is possible only by conduction.

Heat transfer from the solid surface to air [2] is in direct relation to surface friction. The correlation is expressed by the equation

$$Q = \frac{F\, c_p}{v_m} (t_b - t_a) \tag{1}$$

where Q denotes the heat transmitted from 1 cm² of the surface in one second

F „ „ surface friction (tangential force) per cm²

c_p „ „ specific heat of air at constant pressure

v_m „ „ mean velocity of flow

t_b „ „ mean temperature of the surface

t_a „ „ mean temperature of air

It is evident from equation (1) that heat transmission, at a given velocity v_m and temperature difference $t_b - t_a$, depends on surface friction. The value of surface friction is directly related to the mean velocity of air v_m. In the majority of practical cases it is proportional to $\varrho \cdot v_m^2$,

where $\varrho = \dfrac{\gamma}{g}$ denotes the density of air (kg s²/m⁴)

γ „ „ specific gravity of air (kg/m³)

g „ „ gravitational acceleration (m/s²)

Consequently we may write $F = k \cdot \varrho \cdot v_m^2$

where k denotes a constant established by measurement.

By substituting the expression for F in equation (1), heat transmitted per unit of surface is

$$Q = k \cdot \varrho \cdot c_p \cdot v_m \cdot (t_b - t_a) \tag{2}$$

The above expression based upon surface friction represents a safe calculation of transmitted heat. Heat quantities established by practical measurements are invariably higher.

Regardless of the value of the constant k, the relations of friction, velocity, and heat transfer have been established by exact experiments. Surface friction can be expressed in relation to v_m^n, where $n = 1{\cdot}5$ for laminar flow and $n = 2$ for turbulent flow. Heat transmitted can be thus expressed in relation to v_m^{n-1}.

2. Type and thickness of the boundary layer

Several types of flow appear in the boundary layer. Laminar flow with layers sliding one over the other parallel to the solid surface exists closest to the solid body. Movement of molecules normal to the flow direction is negligible, and, therefore, heat transmission in the zone of laminar flow is unsatisfactory. As soon as an irregular transverse movement and eddy diffusion of molecules takes place in the layer, the flow becomes unsteady and is called a turbulent flow. With this type of flow an intensive exchange of molecules takes place between the zone close to the surface and the main stream. Transfer of heat from the solid surface is, therefore, considerably better.

If an air stream is directed against a very thin plate, a boundary layer is formed on the surface of the plate. The thickness and shape of the boundary layer have been studied by numerous authors. Generally the following conclusions have been reached [48]:

The thickness of the boundary layer increases gradually from the leading edge. The flow in the boundary layer is laminar. At a certain distance from the leading edge the thickness of the boundary layer increases suddenly. The abrupt increase of thickness is correlated with the transition of the laminar flow into a turbulent one. The transition occurs at the Reynolds number $\mathrm{Re} = 2$ to 3.10^5.

The Reynolds number is given by the expression $\mathrm{Re} = \dfrac{v_m \cdot l}{\nu}$

where v_m denotes the mean velocity of air (m/s)

l „ „ characteristic linear dimension, in this case the distance from the leading edge (m)

ν „ „ kinematic viscosity of air, $\nu = \dfrac{\eta}{\varrho}$ (m²/s)

η „ „ absolute viscosity of air (kg s/m²)

ϱ „ „ density of air (kg s²/m⁴)

The thickness of the boundary layer is small and depends on the velocity and kinematic viscosity of air. The actual thickness of the boundary layer is difficult to determine, because the velocity gradient decreases gradually. For further considerations in this chapter let us assume the thickness of the boundary layer as being limited by the surface and the layer where the linear velocity $v = 0{\cdot}99\, v_m$.

According to Blasius the thickness of the boundary layer with a laminar flow is given by the relation

$$x_1 = 4{\cdot}5 \left[\frac{l \cdot \nu}{v_m}\right]^{0{\cdot}5}$$

Let us divide both sides by l:

$$\frac{x_1}{l} = 4{\cdot}5 \left(\frac{\nu}{v_m \cdot l}\right)^{0{\cdot}5} = 4{\cdot}5 \left(\frac{v_m \cdot l}{\nu}\right)^{-0{\cdot}5}$$

and write the following equation:

$$\frac{x_1}{l} = 4{\cdot}5\,(\mathrm{Re}_1)^{-0{\cdot}5} \tag{3}$$

The thickness of the boundary layer with a turbulent flow is calculated by a semi-empirical formula stated by von Kármán

$$\frac{x_t}{l} = 0{\cdot}2\,(\mathrm{Re}_1)^{-0{\cdot}15} \tag{4}$$

However, a thin laminar sub-layer exists close to the solid surface in turbulent boundary layers. The thickness of the film does not depend on the distance from the leading edge, and it may be calculated from the formula

$$x_f = \frac{200\,\nu}{v_m} \tag{5}$$

In a turbulent boundary layer air velocity changes rapidly close to the solid surface, and the change becomes gradually slower towards the main air stream. The thickness of the boundary layer from the solid surface up to a layer with a velocity $0{\cdot}8\ V_m$ is thus considerably smaller as shown in Fig. 30, where the total thickness of the boundary layer $v = 0{\cdot}99\ v_m$ is depicted The values of velocities were established by actual measurements on an air foil.

Air velocities were also measured on a metal sheet 0·6 mm thick and 45 mm long. The sheet was coiled into a hollow cylinder which was placed parallel to an air stream flowing at a velocity of 39·5 ft/s (12·2 m/s).

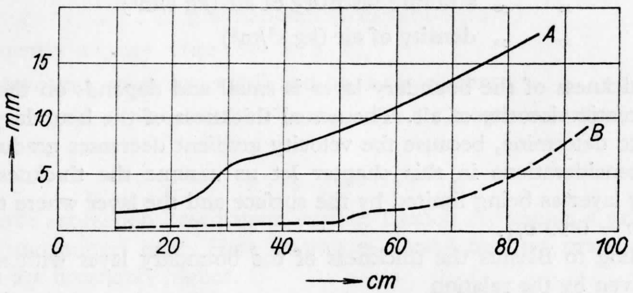

FIG. 30. Thickness of the boundary layer at a velocity of 25 m/s.
A — total thickness of the boundary layer up to $v = 0.99\ v_m$, B — distance of the layer $v = 0.8\ v_m$ from the surface.

Velocities were measured behind the sheet, not on the side, and the results are interesting in showing to what extent air velocity is influenced beyond a metal sheet having approximately the thickness of a cylinder fin of an aircraft engine. The measurements proved, that velocity was reduced in a 3·15 mm high zone. By deducting the sheet thickness of 0·6 mm, the thickness of the boundary layer was established as being 1·275 mm (measured closely behind the sheet). Halfway in the boundary layer, i.e. 0·64 mm from the solid surface, air velocity was reduced by 5% only, and at a distance of 0·3 mm from the surface it was still higher than $0{\cdot}8\ v_m$.

The results obtained by the above measurements are almost in complete agreement with calculated values. According to equation (4) for $l = 45$ mm the thickness of the boundary layer amounts to $x_1 = 1{\cdot}05$ mm; the measured thickness $x_1 = 1{\cdot}10$ mm.

Owing to the fact that heat transfer is more intensive through a turbulent boundary layer, it is important to know at what distance from the leading edge of the fins of an air-cooled engine laminar flow changes to a turbulent one. The sooner such change occurs, the larger the fin surface area exposed to turbulent flow and the better the average heat transfer per unit of the surface area. With thicker fins and air flows oblique to the fin surface, transition from laminar to turbulent flow takes place sooner than according to calculation, and the thickness of the boundary layer at a given distance from the leading edge is larger.

3. Surface friction

Friction on the surface of a thin plate depends on the Reynolds number and also on the type of flow. For laminar flow Blasius stated that the coefficient of surface friction should be calculated from the following equations:

$$\mu_{sl} = 2{\cdot}90\ \mathrm{Re}_x^{-0{\cdot}6} \quad \text{for } 10 < \mathrm{Re}_x < 10^3$$

$$\mu_{sl} = 1{\cdot}328\ \mathrm{Re}_x^{-0{\cdot}5} \quad \text{for } 2\times10^4 < \mathrm{Re}_x < 5\times10^5 \qquad (6)$$

In the case of a turbulent flow, conditions differ mainly in the following respects:

1. Surface friction with turbulent flow is greater than that with laminar flow.
2. Thickness of the boundary layer is greater than in the case of laminar flow and grows more rapidly with the distance l from the leading edge.
3. Velocity increases with the distance from the solid surface more rapidly than in a laminar flow (Fig. 31).

The coefficient of friction in turbulent flow can be calculated from the following formula (von Kármán):

$$\mu_{st} = 0{\cdot}074\ \mathrm{Re}_x^{-0{\cdot}2} - A\,.\,\mathrm{Re}_x^{-1} \qquad (7)$$

where $A = 1700$ for $\mathrm{Re} = 5{\cdot}3\times10^5$

$A = 600$ for $\mathrm{Re} = 1{\cdot}9\times10^5$

$A = 300$ for $\mathrm{Re} = 10^5$

Transition from laminar to turbulent flow depends on the prevailing conditions. The place on a flat plate at which transition occurs is called the transition point, though transition takes place in a certain zone rather than in a point. The transition point determined by the Re value depends on the smoothness of the surface, the turbulence of the streaming air, the angle of the air stream, etc.; the corresponding range of Re numbers is $\mathrm{Re} = 2\times10^5$ to 2×10^6.

For a given plate the Reynolds number, $\mathrm{Re} = \dfrac{v_m\,.\,l}{\nu}$, of the transition point remains constant. Therefore, the distance l of the transition point from the leading edge decreases with increasing velocity v_m. This means also that a change of flow velocity causes an alteration of the ratio of areas exposed to laminar and turbulent

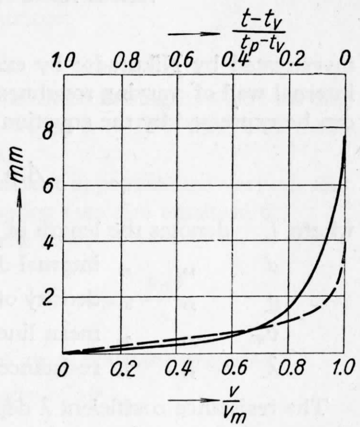

FIG. 31. Velocity and temperature gradient in the boundary layer.
Dashed curve — laminar flow, full curve — turbulent flow.

flow. The position of the transition curve depends on the degree of turbulence of the free flow. In a flow without turbulence a high value of Re is critical, and, on the contrary, in a turbulent flow a low value of Re is critical. The transition from laminar to turbulent flow occurs at the critical Re value.

Surface friction is strongly influenced by the unevenness of the surface. The correlation between surface friction and unevenness of the surface was

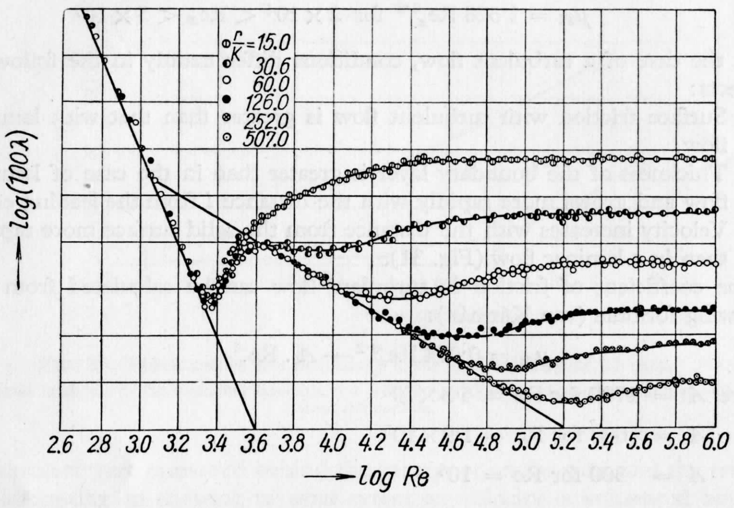

FIG. 32. Relationship between coefficient of resistance and Reynolds number for various values of relative surface roughness.

investigated by Nikuradse by examining an air stream in a tube having an internal wall of varying roughness. The loss of pressure head in the test tube can be expressed by the equation:

$$\Delta p = \frac{\lambda\, l}{d}\,\frac{\varrho}{2}\, v_m^2 \qquad (8)$$

where l denotes the length of the tube (m)

d „ „ internal diameter of the tube (m)

ϱ „ „ density of air (kg s²/m⁴)

v_m „ „ mean linear velocity of the air (m/s)

λ „ „ resistance coefficient

The resistance coefficient λ depends not only on the Reynolds number but also on relative roughness of the surface

$$R_r = \frac{r}{k}$$

where r denotes the radius of the tube and

K „ „ mean height of projections

This correlation is illustrated in Fig. 32 where the values of log (100 λ) are plotted against log Re for various values of relative roughness R_r.

The diagram shows that in a laminar flow up to log Re = 3·4 surface friction is not influenced by the surface roughness, because projections of the uneven wall are covered by the laminar boundary layer. If the flow becomes turbulent, the influence of roughness, with projections protruding from the laminar layer, is being felt progressively with an increasing Re. At Re = = 100,000 and larger the boundary layer is reduced to a thin film covering only projections of $R_r = 507$ or smaller.

As mentioned before, transmission of heat from the surface depends to a very large extent on surface friction. Theoretically, the quantity of heat Q' transferred from a plate of a width b and length l with a laminar flow on the surface, can be computed from the equation

$$\frac{Q'}{b\,.\,k\,.\,(t_b - t_a)} = \mu_{sl}\,\frac{v_m l}{\nu}$$

Measurements carried out on a very thin plate having a width b and length l with a laminar flow on the surface have shown, that the quantity of heat actually transferred is given by the relation

$$\frac{Q'}{b\,.\,k\,.\,(t_b - t_a)} = 1{\cdot}50\left(\frac{v_m\,.\,l}{\nu}\right)^{-0\cdot5} \qquad (9)$$

where k denotes the thermal conductivity of air

t_b „ „ temperature of the surface

t_a „ „ temperature of air

The tests were carried out with plates of four different lengths with l varying within the range 12·7 to 3·3 mm constant width b = 12·7 mm and mean linear velocities varying from 4·8 to 0·75 m/s.

The constant 1.50 established by measurement approximates very closely the theoretical constant in the preceding equation (see also equation 6).

The following equation is valid for turbulent flow:

$$\frac{Q'}{b\,.\,c_p\,.\,(t_b - t_a)} = \mu_{st}\,.\,\varrho\,.\,v_m\,.\,l = \eta\,.\,\mu_{st}\,\frac{v_m l}{\nu} \qquad (10)$$

where c_p denotes the specific heat of air at constant pressure

$\eta = \nu\,.\,\varrho$ „ „ absolute viscosity of air

The constants μ_{sl} and μ_{st} in the respective equations depend upon the Reynolds number; for laminar flow $\mu_{sl} \propto Re^{-0\cdot5}$ and for turbulent flow $\mu_{st} \propto Re^{-0\cdot15}$. Therefore, at a given mean velocity v_m surface friction is more regular in the case of a turbulent flow, because μ_s changes with the distance l

more slowly than in the case of a laminar flow. This consideration holds for a very thin plate placed exactly parallel to the flow direction.

For cooling fins of air-cooled engines it is important to know the point at which the transition from laminar to turbulent flow occurs. Fins should be spaced so as to avoid any noticeable overlap between the boundary layers of two opposing fins.

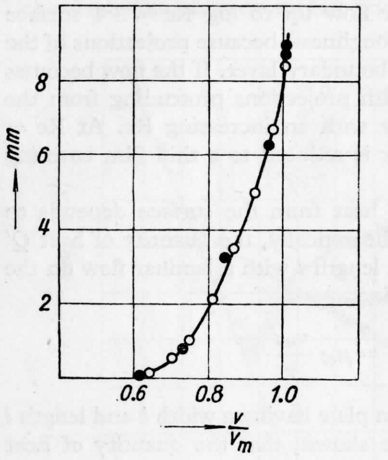

FIG. 33. Velocity and temperature gradient in the boundary layer.

As stated previously, heat transfer is correlated with the tangential force F acting on the surface (surface friction). This assumption has been confirmed by measurements of the temperature difference in the boundary layer, according to which temperature difference is in complete harmony with the velocity gradient. Results of measurements of temperature and velocity in the boundary layer are shown in Fig. 33. Full dots represent velocities, while temperatures are shown by circles. It can be seen that measurement results of both values are in complete conformity.

At the leading edge of a fin exposed to an air stream the thickness of the boundary layer is small, and the flow is laminar. Heat is transferred mainly by conduction and with increasing thickness of the boundary layer heat transmission becomes worse.

The afore described analysis of flow types leads to the following conclusion: heat transfer from a solid surface is, in spite of the greater thickness of the boundary layer, better in a turbulent flow than through the thinner boundary layer of a laminar flow.

The influence of surface roughness on heat transfer was tested on a cylinder of an air-cooled engine. The surface of the fins was roughened by punches 0.5 mm deep and 1·8 mm apart. No substantial difference in heat transfer was found as compared with a fin of smooth surface. It was also proved that, contrary to previous expectations, the rapid vibration of fins of an air-cooled engine had no influence on heat transfer.

4. The surface coefficient of heat transfer

The quantity of heat transferred from a solid surface to a surrounding gas is determined by the basic equation:

$$Q = q \cdot \Delta t \cdot A$$

where A denotes the area of the cooling surface (m^2)

Δt „ „ temperature difference between surface and gas (°C)

q „ „ surface coefficient of heat transfer (kcal/m^2 h °C)

The surface coefficient of heat transfer from wall to air, or for a reversed transfer, depends upon the velocity of air, specific pressure, specific weight, specific heat and viscosity of air, the roughness of the surface and absolute temperature. Therefore, it cannot be expressed by a simple formula. Some approximate formulas are used in the following sections.

Exhaustive measurements of the surface coefficient of heat transfer were carried out on an electrically heated cylinder with copper fins having a diameter of 114·3 mm. In this case the surface coefficient of heat transfer q was found to comply with the following formula:

$$q = 1{\cdot}18\,(1 + 0{\cdot}0075\ T_m)\ v_m^{0{\cdot}73} \tag{11}$$

where T_m denotes the arithmetic mean of absolute temperature of the fin surface and air, i.e. the absolute mean temperature of the boundary layer (°C)

v_m „ „ mean air velocity (km/h).

Measurements were made at fin temperatures ranging from 30 °C to 155 °C and air temperatures from 15 to 18 °C. Internal diameter of fins amounted to 114 mm, outer diameter 146 mm, thickness of fins 0·55 mm and spacing 8 mm.

The correlation of the surface coefficient of heat transfer and absolute temperature was established by measurements at a constant air temperature of 15 °C and fin temperatures of 105 and 155 °C. The corresponding absolute mean temperatures are 333 and 358 °C. The increase of the temperature by 50 °C meant an increase of the temperature factor (parenthesized term in equation 11) from 3·50 to 3·68, i.e. by 5% which was exactly the relation by which the quantity of heat transferred was increased.

Fig. 34 shows values of the surface coefficient of heat transfer obtained by measurements on an even surface 152 × 152 mm having a temperature of 65 °C and exposed to an air stream of 20 °C. In this case the transferred heat was proportional to the 0·725th power of the mean air velocity v_m.

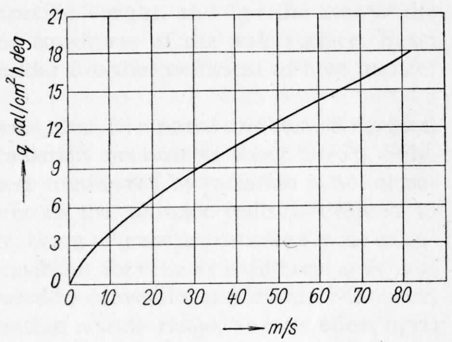

FIG. 34. Surface coefficient of heat transfer measured on an even surface in relation to air velocity.

Other measurements on even fins 162 mm long gave results of coefficients being proportional to $v_m^{0{\cdot}747}$.

Thus the value $v_m^{0{\cdot}73}$ can be regarded as an entirely satisfactory average applicable in calculations of normal cylinders of air-cooled engines. It should be borne in mind that the heat transfer coefficient for laminar flow is proportional to $v_m^{0 \cdot 5}$, for turbulent flow to v_m. In the case of fins 152 mm long, exposed to an air stream having a velocity of 65 to 213 ft/s (20—65 m/s), the surface area exposed to laminar flow varies between $^1/_4$ and $^3/_4$ of the total surface area, and therefore, the exponent 0·73 can be regarded as a satisfactory compromise.